Determine if the following integrals converge or diverge. If they converge, compute them.

(a) $\int_0^{\infty} \cos x \, dx$ (c) $\int_{-1}^{1} x^{-2/3} \, dx$

(b) $\int_0^1 \frac{\ln x}{x^{1/2}} \, dx$

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(a) $\int_0^{\infty} \cos x \, dx$

$= \lim_{N \to \infty} - \sin x \Big|_0^{\infty}$

$\therefore$ Diverge.

(b) $\int_0^1 \frac{\ln x}{x^{1/2}} \, dx$

$= \int_0^1 x^{-1/2} \ln x \, dx$

$= \ln x \int x^{-1/2} \, dx \Big|_0^1 - \int_0^1 \frac{1}{x} \int x^{-1/2} \, dx$

$= \ln x \cdot \left(\frac{x^{1/2}}{1/2} \right) \Big|_0^1 - \int_0^1 \frac{1}{x} \cdot 2x^{1/2} \, dx$

$$= 2\ln x \cdot x^{1/2} \Big|_0^1 - 2\int_0^1 x^{-1/2}\, dx$$

$$= -2 \lim_{N\to 0} \frac{\ln N}{-1/N^{1/2}} - 2\cdot 2\, x^{1/2} \Big|_0^1$$

$$= -2 \lim_{N\to 0} \frac{1/N}{1/2 \cdot 1/N^{3/2}} - 4$$

$$= -2 \lim_{N\to 0} \frac{2N^{3/2}}{N} - 4$$

$$= -4 \lim_{N\to 0} N^{1/2} - 4$$

$$= -4$$

(c) $$\int_{-1}^{1} x^{-2/3}\, dx$$

$$= \int_0^1 x^{-2/3}\, dx + \int_{-1}^{0} x^{-2/3}\, dx$$

$$= 3x^{1/3}\Big|_0^1 + 3x^{-2/3}\Big|_{-1}^{0}$$

$$= 3(1) + 3(1)$$

$$= 6$$